3.5.1. Semantic Problems:
Tautology, Contradiction, Logical Equivalence, and Validity

A. **Translate** each English sentence into the formal language and build a **truth table** for that sentence. On the basis of that truth table, find a **simpler English sentence** that is **logically equivalent** to the original.

1. Rex got a royalty check if, and only if, he published if and only if he published.

> (*For Problems 2 through 5, the simpler sentence won't appear as an earlier step in the truth table.*)

2. Assuming that Trixie won the poker tournament if and only if she's buying a hot tub, she won the poker tournament.

3. Trixie won the poker tournament if, and only if, she's buying a hot tub if she won the poker tournament.

4. Trixie won the poker tournament if, and only if, she's buying a hot tub only if she won the poker tournament.

5. Neko went hungry if and only if neither she nor Suki went hungry.

(*For Problems 6 through 9, try to find a simpler **conditional** – it won't appear as an earlier step in the truth table. Use the **same translation key** for all four problems.*)

6. If Dick wants a martini, then he wants one if and only if Dora wants one.

7. Dick wants a martini if and only if both he and Dora want a martini.

8. Dora wants a martini if and only if either she or Dick wants one.

9. Either Dora wants a martini, or Dick wants one if and only if Dora wants one.

B. **Translate** each of the following sentences into the formal language; then use a **truth table** or **truth trees** to decide whether that sentence is a **tautology**, a **contradiction**, or **neither**.

1. If Suki’s going to Las Vegas then she’s going to Las Vegas.

2. If Suki’s going to Las Vegas then she’s not going to Las Vegas.

3. If Suki’s either lecturing on microblading or not lecturing on microblading then she’s going to Las Vegas.

4. If Suki’s going to Las Vegas then she’s either lecturing on microblading or not lecturing on microblading.

5. If Suki’s lecturing on microblading then she’s going to Las Vegas without going to Las Vegas.

6. If Suki’s either lecturing on microblading or not lecturing on microblading then she’s going to Las Vegas without going to Las Vegas.

7. Rex taught Business Logic if he taught Business Logic; otherwise he didn’t.

8. If Jake’s playing pool, then assuming he’s happy he’s happy.

9. If Jake’s playing pool, then assuming he’s happy he’s playing pool.

10. If Jake’s playing pool, then assuming he’s playing pool he’s happy.

11. Lucretia took her umbrella if she went out, though she went out without taking her umbrella.

12. If Letitia didn't skip class and she also passed the quiz, then she passed the quiz.[1]

13. If Letitia didn't skip class and also pass the quiz, then she passed the quiz.[1]

14. Dr. Slim got sued if and only if he got sued.

15. It's not the case that: Dr. Slim got sued if and only if he got sued.

16. Dr. Slim got sued if and only if he didn't get sued.

17. It's not the case that: Dr. Slim got sued if and only if he didn't get sued.

18. Dr. Slim didn't get sued if and only if he didn't get sued.

19. Assuming Jake's unhappy only if Jezebel is, if Jezebel's happy then so is Jake.

20. Assuming Jake's unhappy only if Jezebel is, Jezebel's happy if Jake is.

21. If Sentence (21) is false then it isn't false, but if it isn't false then it *is* false.

(*Adapting* Gamut 1982/1991 vol. I: 141, Problem 10e)

[1] See 2.9 §4 on how deleted repetition affects connective scope.

C. **Translate** each of the following arguments into the formal language; then use **truth tables** or a **truth tree** to decide if the argument is **valid**.

1. If Suki’s ticket is valid, then so is mine. My ticket’s invalid. ∴ Neither Suki’s ticket nor mine are valid.

2. If my ticket’s invalid then so is Suki’s. ∴ If Suki’s ticket is valid then so is mine.

3. Provided she studied, Letitia passed Business Logic. Letitia passed Business Logic only if she studied. ∴ Letitia studied, and she passed Business Logic.

4. Either the client’s still in a therapy session or Dr. Slim’s having a drink. If the client’s still in a therapy session, Dr. Slim’s having a drink. ∴ Dr. Slim’s having a drink.

5. Dr. Slim’s going to Reno if Kitty is, and Kitty’s going to Reno. ∴ Both Kitty and Dr. Slim are going to Reno.

6. Trixie will play blackjack if Elvis does. Trixie will play blackjack (even) if Elvis doesn’t. ∴ Trixie will play blackjack.

7. Trixie will play blackjack only if Elvis does. Trixie will play blackjack only if Elvis doesn’t. ∴ Trixie won’t play blackjack.

8. Either Suki will order a California roll or Neko will. Neko will order a California roll if Suki does. ∴ Both Suki and Neko will order a California roll.

9. Either Suki will order a California roll or Neko will. Neko will order a California roll if and only if Suki does. ∴ Both Suki and Neko will order a California roll.

10. If Dick knows who poisoned the gin then Dora does too. Dora doesn’t know who poisoned the gin unless Dick knows. ∴ Both Dick and Dora know who poisoned the gin.

11. Letitia won't pass Business Logic without studying. ∴ Letitia will pass Business Logic only if she studies.[2]

12. If Dr. Slim isn't a physician, he's not a physician who performs surgery. Dr. Slim isn't a physician. ∴ Dr. Slim doesn't perform surgery. [3]

13. If Dr. Slim is a physician, he's one who doesn't perform surgery. Dr. Slim isn't a physician. ∴ Dr. Slim doesn't perform surgery.

14. If Dr. Slim is a physician, he's one who doesn't perform surgery. ∴ Dr. Slim's not a physician who performs surgery.[3]

15. Dr. Slim's not a physician who performs surgery. ∴ If Dr. Slim is a physician, he's one who doesn't perform surgery.[3]

16. Dick will have a Pimm's Cup if Dora has one; otherwise he won't. ∴ Either both Dick and Dora will have a Pimm's Cup, or neither of them will.

17. Lucretia went clubbing at Novo if she finished her lab report; otherwise she didn't. ∴ Lucretia went clubbing at Novo if and only if she finished her lab report.

18. Suki got an A if she passed the quiz; otherwise she got a B. Suki didn't get a B. ∴ Suki passed the quiz and got an A.

19. If Jack was arrested for scaling a skyscraper he's not running with the bulls in Pamplona; otherwise he is. ∴ Either Jack was arrested for scaling a skyscraper or he's running with the bulls in Pamplona, but not both.

20. Kitty has a kong of flowers only if she has a joker, assuming she has a kong of flowers. She doesn't have a kong of flowers if she doesn't have a joker. ∴ Kitty has a joker.

21. It's Thursday, assuming that if it's Thursday then Jack's surfing. ∴ Jack's surfing, provided that it's Thursday only if he's surfing.

---

[2] See 2.10, §3 on the negation of a "without" sentence.

[3] See 2.10, §3 on the negation of a sentence with a relative clause.

22. Assuming Lucretia's a goth only if Letitia's a goth, Letitia's a goth. Letitia's a goth only if she's not a goth. ∴ Provided that Letitia's a goth if Lucretia's a goth, Lucretia's a goth.

23. Jake is both honest and responsible if he's a member of the Surf Club. Jake isn't a member of the Surf Club. ∴ Jake's either dishonest or irresponsible.

(*Adapted from* Kleene 1967/2002: 66, #14.1a)

24. Jake's both honest and responsible if and only if he's a member of the Surf Club. Jake isn't a member of the Surf Club. ∴ Jake's either dishonest or irresponsible.

25. If Letitia liked the movie, it had a happy ending. If Lucretia liked the movie it didn't have a happy ending. ∴ Either Letitia liked the movie or Lucretia did, but not both.

26. If Letitia liked the movie, it had a happy ending. If Lucretia liked the movie it didn't have a happy ending. ∴ Letitia and Lucretia didn't both like the movie.

27. If both Dick and Dora ordered a Pink Squirrel then so did Trixie. ∴ Dick ordered a Pink Squirrel, and if Dora did too then so did Trixie.

28. Dick ordered a Pink Squirrel, and if Dora did too then so did Trixie. ∴ If both Dick and Dora ordered a Pink Squirrel then so did Trixie.

29. Kitty will have both a manicure and a massage if the check clears, and a manicure without a massage otherwise. ∴ Kitty will have a manicure, and she'll have a massage if and only if the check clears.

30. The president will issue an executive order if the bill stalls in either the House or the Senate. The Logic lobby will mobilize only if the bill stalls in the Senate. Assuming Boolean PAC holds a phone campaign, the bill will stall in the House. Provided Boolean PAC doesn't hold a phone campaign, the Logic lobby will mobilize. ∴ The president will issue an executive order.

31. If God exists, then S/he's omnipotent, omniscient, and benevolent. If God is omniscient, then S/he knows that evil exists if and only if it does exist. If God is omnipotent, S/he can prevent evil. Provided God can prevent evil and knows that evil exists, S/he's not benevolent if s/he doesn't prevent it. Evil doesn't exist if God prevents it. Evil exists. ∴ God does not exist.

(*Adapted from* Kalish, Montague, and Mar 1980: 35, Problem 35)

32. If the bartender is the killer then Dick will catch her in a lie, assuming Dora joins the conversation. Provided that Dick will catch the bartender in a lie if the bartender is the killer, the bartender will confess to the crime. The bartender will confess to the crime only if she's the killer. ∴ If Dora joins the conversation, Dick will catch the bartender in a lie.

33. If the bartender didn't kill the baron, then either the sommelier or the bootlegger did. The merlot was poisoned provided the sommelier killed the baron. There was antifreeze in the sour mash if the bootlegger killed the baron. The merlot wasn't poisoned, and the bartender didn't kill the baron. ∴ The bootlegger killed the baron.

(*Adapted from* Partee, ter Meulen, and Wall 1990: 134, Problem 10a.)

34. That consonantal segment is prevocalic if it occurs initially; otherwise it's voiceless. Provided it's either prevocalic or voiceless, it's both continuant and strident. Assuming it's continuant, it's tense if it's strident. If it's tense, then if it occurs initially it's palatalized. ∴ That consonantal segment is palatalized and voiceless.

(*Adapted from* Partee, ter Meulen, and Wall 1990: 134, Problem 10e)

35. If we have either ice cream or cake, then either we'll have ice cream without having pie or we'll have both brownies and sherbet. We'll have cake and brownies but we won't have both pie and fudge. Unless we have pie without having fudge, we'll have neither brownies nor sherbet. ∴ Either we'll have sherbet without having ice cream, or we'll have fudge without having ice cream.[4]

[4] This argument first appeared in 1.13. Warning: a truth table for this argument takes 1,625 steps (counting each sentence, 1, and 0 as a step).

D. Build a **truth table** or **truth tree** for each of the following sentences to show that the sentence is a **tautology**.

T3.1. $(P \rightarrow P)$

T3.2. $((P \rightarrow \sim P) \rightarrow \sim(\sim P \rightarrow P))$

T3.3. $((P \rightarrow Q) \rightarrow ((Q \rightarrow R) \rightarrow (P \rightarrow R)))$

T3.4. $(P \rightarrow (\sim P \rightarrow Q))$

T3.5. $(P \rightarrow ((P \rightarrow Q) \rightarrow Q))$

T3.6. $(((P \rightarrow Q) \rightarrow P) \rightarrow P)$

T 3.7. $(((P \rightarrow Q) \rightarrow P) \leftrightarrow P)$

T 3.7a. $(((P \rightarrow Q) \rightarrow P) \rightarrow P)$

T 3.7b. $(P \rightarrow ((P \rightarrow Q) \rightarrow P))$

T 3.8. $(((P \rightarrow Q) \leftrightarrow (P \rightarrow (P \wedge Q))$

T 3.8a. $((P \rightarrow Q) \rightarrow (P \rightarrow (P \wedge Q)))$

T 3.8b. $((P \rightarrow (P \wedge Q)) \rightarrow (P \rightarrow Q))$

T. 3.9. $((P \rightarrow Q) \vee (Q \rightarrow \sim R))$

T. 3.10. $((P \rightarrow R) \vee (Q \rightarrow \sim R))$

T3.10. $((P \rightarrow Q) \rightarrow ((P \vee R) \rightarrow (Q \vee R)))$

T3.10. $(((P \rightarrow Q) \wedge (R \rightarrow S)) \rightarrow ((P \wedge R) \rightarrow (Q \wedge S)))$

T3.11. $((P \rightarrow (Q \rightarrow R)) \leftrightarrow ((P \wedge Q) \rightarrow R))$

T3.11a. $((P \rightarrow (Q \rightarrow R)) \rightarrow ((P \wedge Q) \rightarrow R))$

T3.11b. $((P \rightarrow (Q \rightarrow R)) \rightarrow ((P \wedge Q) \rightarrow R))$

T3.12. $((P \rightarrow (Q \rightarrow R)) \leftrightarrow (Q \rightarrow (P \rightarrow R)))$

T3.12a. $((P \rightarrow (Q \rightarrow R)) \rightarrow (Q \rightarrow (P \rightarrow R)))$

T3.12b. $((Q \rightarrow (P \rightarrow R)) \rightarrow (P \rightarrow (Q \rightarrow R)))$

T3.13. $((P \rightarrow Q) \leftrightarrow (\sim P \vee Q))$

T3.13a. $((P \rightarrow Q) \rightarrow (\sim P \vee Q))$

T3.13b. $((\sim P \vee Q) \rightarrow (P \rightarrow Q))$

T3.14. $((P \rightarrow Q) \leftrightarrow \sim(P \wedge \sim Q))$

T3.14a. $((P \rightarrow Q) \rightarrow \sim(P \wedge \sim Q))$

T3.14b. $(\sim(P \wedge \sim Q) \rightarrow (P \rightarrow Q))$

T3.15. $((P \rightarrow \sim P) \leftrightarrow \sim P)$

T3.15a. $((P \rightarrow \sim P) \rightarrow \sim P)$

T3.15b. $(\sim P \rightarrow (P \rightarrow \sim P))$

T3.16. $((P \rightarrow (Q \wedge \sim Q)) \leftrightarrow \sim P)$

T3.16a. $((P \rightarrow (Q \wedge \sim Q)) \rightarrow \sim P)$

T3.16b. $(\sim P \rightarrow (P \rightarrow (Q \wedge \sim Q)))$

T3.17. $(P \leftrightarrow P)$

T3.18. $\sim(P \leftrightarrow \sim P)$

T3.19. $(\sim(P \leftrightarrow Q) \leftrightarrow (P \leftrightarrow \sim Q))$

T3.19a. $(\sim(P \leftrightarrow Q) \rightarrow (P \leftrightarrow \sim Q))$

T3.19b. $((P \leftrightarrow \sim Q) \rightarrow \sim(P \leftrightarrow Q))$

T3.20. $((P \leftrightarrow (Q \leftrightarrow Q)) \leftrightarrow P)$

T3.20a. $((P \leftrightarrow (Q \leftrightarrow Q)) \rightarrow P)$

T3.20b. $(P \rightarrow (P \leftrightarrow (Q \leftrightarrow Q)))$

T3.20c. $((P \rightarrow (Q \leftrightarrow Q)) \rightarrow P)$

T3.21. $(\ (P \leftrightarrow (Q \leftrightarrow \sim Q)) \leftrightarrow \sim P \ )$

T3.21a. $(\ (P \leftrightarrow (Q \leftrightarrow \sim Q)) \rightarrow \sim P \ )$

T3.21b. $(\ \sim P \rightarrow (P \leftrightarrow (Q \leftrightarrow \sim Q)) \ )$

T3.21c. $(\ (P \rightarrow (Q \leftrightarrow \sim Q)) \rightarrow \sim P \ )$

T3.22. $(\ (P \leftrightarrow Q) \leftrightarrow (Q \leftrightarrow P) \ )$

T3.23. $(\ (P \leftrightarrow Q) \leftrightarrow R) \leftrightarrow (P \leftrightarrow (Q \leftrightarrow R) \ )$

T3.24. $(\ (P \leftrightarrow Q) \leftrightarrow ((P \rightarrow Q) \wedge (Q \rightarrow P)) \ )$

T3.24a. $(\ (P \leftrightarrow Q) \rightarrow ((P \rightarrow Q) \wedge (Q \rightarrow P)) \ )$

T3.24b. $(\ ((P \rightarrow Q) \wedge (Q \rightarrow P)) \rightarrow (P \leftrightarrow Q) \ )$

T3.25. $(\ (P \leftrightarrow Q) \leftrightarrow ((P \wedge Q) \vee (\sim P \wedge \sim Q)) \ )$

T3.25a. $(\ (P \leftrightarrow Q) \rightarrow ((P \wedge Q) \vee (\sim P \wedge \sim Q)) \ )$

T3.25b. $(\ ((P \wedge Q) \vee (\sim P \wedge \sim Q)) \rightarrow (P \leftrightarrow Q) \ )$

T3.26. $(\ ((P \leftrightarrow (Q \rightarrow R)) \rightarrow ((P \leftrightarrow Q) \rightarrow R) \ )$

T3.27. $(\ ((P \wedge (Q \rightarrow R)) \rightarrow ((P \wedge Q) \rightarrow R) \ )$

T3.28. $(\ ((P \vee Q) \rightarrow R) \rightarrow ((P \vee (Q \rightarrow R)) \ )$